Computation of Hypersonic Turbulent Boundary Layers with Heat Transfer

J. S. SHANG*
Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio

Nomenclature

Pr_t = turbulent Prandtl number, Eq. (3)
T = static temperature
u, v = streamwise and normal velocity components
ε = kinematic eddy viscosity coefficient
λ_t = apparent turbulent thermal conductivity
ρ = density
$\langle\ \rangle$ = denotes time-mean average
$'$ = denotes the fluctuating property

Theme

THE concept of eddy viscosity has been applied to predict turbulent boundary layers in hypersonic flow regions.[1,2] Hopkins et al.[2] found that the multilayer viscosity model underpredicts the skin-friction coefficient by around 10% for hypersonic flow conditions. The neglected density fluctuation terms in the Reynolds stress were identified as the cause of this deficiency. Recent experimental investigations[3] on turbulent boundary layers revealed that the turbulent transport of energy decreases more rapidly toward the wall than the momentum transport. Their results seem to indicate that the heat transfer and temperature profile of turbulent flows cannot be ascertained by an oversimplified constant turbulent Prandtl number. At hypersonic Mach numbers, experimental data[4] also recorded a distinct normal pressure gradient within the turbulent boundary layer. At the present time, no analytical method or systematic evaluation has been attempted to include these phenomena. The present analysis intends to determine the significance of the aforementioned phenomena and also to provide suitable means to improve the numerical prediction scheme for hypersonic turbulent boundary layers.

Content

The explicit dependence of the density fluctuation is introduced into the eddy viscosity model by incorporating the data of Kistler.[5] The temperature fluctuations obtained by Kistler were based on the assumption that the pressure fluctuation is negligible. Thus the temperature and density fluctuation are identical in magnitude but opposite in sign. The justification to use his data at higher Mach numbers as a valid modification is based upon the fact that the relative temperature fluctuation when scaled by $2(T_o - T_\infty)/(T_o + T_\infty)$ is almost independent of Mach number. The temperature fluctuation data obtained by Laderman and Demetriades for a highly cooled wall at a Mach number of 9.37 was similar to the data of Kistler although at a lower value. These fluctuation terms contained in the mean equation of motion can be approximated as

$$\langle(\rho v)'u'\rangle \simeq \rho\langle u'v'\rangle\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right] = \rho\varepsilon\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right]\left(\frac{\partial u}{\partial y}\right) \quad (1)$$

$$\langle(\rho v)'T'\rangle \simeq \rho\langle v'T'\rangle\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right] = \rho\lambda_t\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right]\left(\frac{\partial t}{\partial y}\right) \quad (2)$$

$$Pr_t = [\langle u'v'\rangle(\partial t/\partial y)]/[\langle v'T'\rangle(\partial u/\partial y)] \quad (3)$$

The effect of the variable turbulent Prandtl number was studied by using analytical descriptions of the uncertainty envelope of the turbulent Prandtl number.[6] This criterion was established originally for incompressible flows but also found to be valid for compressible flows. The normal pressure gradient in the boundary layer due to turbulence was taken into account by including the y momentum equation of mean motion. The normal momentum equation indicates that the variation of the static pressure is small; however, the combination of the pressure and the normal component of the Reynolds shear stress remains invariant across the turbulent boundary layer

$$(\partial/\partial y)[\langle(\rho v)'v'\rangle + p] = 0 \quad (4)$$

The fluctuation term $\langle(\rho v)'v'\rangle$ in the normal momentum equation was correlated with the Reynolds shear stress $\langle(\rho v')v'\rangle$ to predict the pressure variation within the boundary layer

$$\langle(\rho v)'v'\rangle \simeq \rho\langle v'v'\rangle\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right] = 1.4\rho\varepsilon\left[1 + \frac{(\rho'^2)^{1/2}}{\rho}\right]\left(\frac{\partial u}{\partial y}\right) \quad (5)$$

The numerical scheme used in the present analysis was identical to Ref. 7 except the density variation was deduced from the equation of state by the computed temperature and pressure.

Figure 1 presents the static pressure variation across the turbulent layer at Mach number of 9.37. The experimental data were obtained by Laderman and Demetriades.[4] The calculated

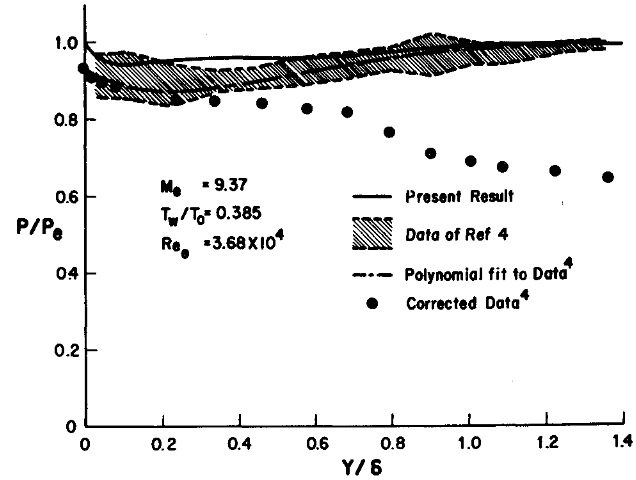

Fig. 1 Static pressure distribution across turbulent boundary layer.

Presented as Paper 73-699 at the AIAA 6th Fluid and Plasma Dynamics Conference, Palm Springs, Calif., July 16–18, 1973; submitted July 16, 1973; synoptic received December 13, 1973. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $1.00; hard copy, $5.00. **Order must be accompanied by remittance.**

Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

* Aerospace Engineer, Hypersonic Research Laboratory. Member AIAA.

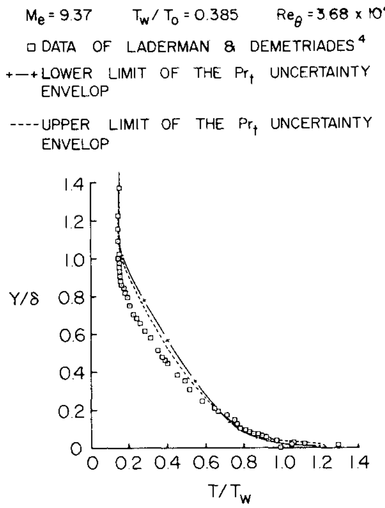

Fig. 2 Effect of turbulent Prandtl number on temperature profile.

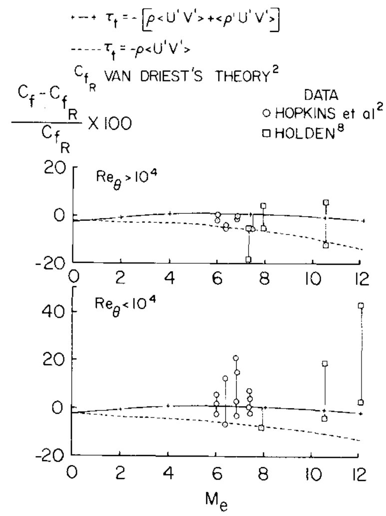

Fig. 3 Skin-friction coefficient variation with Mach number.

pressure distribution within the boundary layer agrees very well with the uncorrected measurement. The pressure decreases very rapidly from the wall and reaches the minimum value near the outer edge of the sublayer, then approaches the freestream value asymptotically. The pressure variation across the turbulent boundary layer is directly proportional to the turbulent shear stress $\langle(\rho v)'u'\rangle$. The relative magnitude of the pressure variation is proportional to the square of the freestream Mach number and the correlation function between $\langle v'^2\rangle$ and the turbulent kinetic energy. The present modeling of the pressure variation assumed no freestream turbulence, therefore, the surface pressure equals the freestream value. If the freestream turbulence value can be assessed, the maximum pressure would be attained at the surface.

The influence of turbulent Prandtl number on the detailed flowfield was investigated for two different turbulent Prandtl number distributions. Both Pr_t distributions were recovered from the limits of the Pr_t uncertainty envelope. The computed velocity profiles are nearly identical, thus only the static temperature profiles are presented in Fig. 2. The agreement is fair between the data and the computed results. One observes that the calculations predict correctly the location of the maximum temperature in the turbulent boundary layer. The difference betwen the two solutions of the limiting Prandtl number distributions is small. Significant differences between the two solutions appear only in the magnitude of the maximum temperature and in the law of the wake region. The solution associated with the upper limit Pr_t variation yields a lower value of C_f (-3.2%) than the solution by the lower limit Pr_t. However, the trend is reversed in predicting the Stanton number ($+3\%$). In all, the solutions appear to be insensitive to the variation of the turbulent Prandtl number.

A summary of the eddy viscosity model with the density fluctuation correction over a wide range of the freestream Mach number is present in Fig. 3. Van Driest's theory suggested by Hopkins et al. is adopted for use in the criterion. The comparison with data[2,8] is divided into two groups. The first group is restricted to Re_θ less than 10^4, while the second group is presented for Re_θ greater than 10^4. The difference between solutions with and without the density fluctuation correction diminishes as the freestream Mach decreases. This behavior is expected because the density fluctuation correction is proportional to $(T_o - T_\infty)$. The difference between the Van Driest theory and the present calculation with density fluctuation is small. The agreement between data and the calculations by the eddy viscosity model with the density fluctuation correction is excellent except in the low Re_θ and high Mach number region. The eddy viscosity model with the density fluctuation correction exhibits an observable improvement (up to 10%) over the eddy viscosity model without the density fluctuation correction.

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